

Master QFin, CTFI
Midterm Exam Wed

Hints

- This is an open book exam; no pocket calculators etc.
- Please mark every sheet you upload with your name and Mat.Nr.
- By uploading your solution you implicitly state that you solved the assignment completely on your own.
- Good luck !

1. Brownian motion and martingales (2 points.) Let $W_t, t \geq 0$, be standard Brownian motion and define $\mathcal{F}_t := \sigma(W_s, s \leq t)$. Show that W_t is a martingale with respect to the filtration $\{\mathcal{F}_t\}$.

2. Application of the Ito formula (4 points) Let B be Brownian motion. Use Ito's formula to compute the semimartingale decomposition of $X_t = B_t^3$ and compute $[X]_t$

3. (6 points) Consider a twice differentiable function F on \mathbb{R} with bounded first derivative F' and a Brownian motion W .

- Use the Ito formula to show that $M_t = F(W_t) - \frac{1}{2} \int_0^t F''(W_s) ds$ is a local martingale. (3 points)
- Compute $[M]_t$ and show that $E([M]_t) < \infty$ (so that M is a true martingale). (3 points)